the computed and the experimentally measured mode shapes was made for all of the frequencies and good agreement was obtained.

IV. Discussion and Conclusions

It is concluded that the theoretical model as described herein predicts frequencies and mode shapes that are in good agreement with experiment.

It has been observed that accurate prediction of higher mode frequencies, for plates of the type discussed here, often requires inclusion of the effects of rotary inertia of attached masses in the analysis. This is not surprising as the masses will undergo harmonic rotation for some of the higher modes. Extension of the analysis to include these rotary inertia effects is currently underway.

To the authors' knowledge, the work reported here represents the first comprehensive analytical and experimental study of this interesting and timely problem. The analysis provides the designer with a powerful tool for optimizing the distribution of rigid point supports and attached masses on the plate surface. The experimental data will provide other researchers with reference points against which they can compare their theoretical results.

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Objective Functions for the Nonlinear Curve Fit of Frequency Response Functions

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Introduction

THE improvement of finite element models of structures based on experimental data is usually referred to as model updating. Finite element model updating methods in structural dynamic problems generally consist of making corrections of the theoretical stiffness and mass matrices so that the eigenvalues and eigenvectors of the finite element model get closer to the experimental values.[1] The number of parameters in the finite element model is normally too big, and constraint equations[2] or localization methods[3] must be used to reduce the number of parameters of the theoretical model that must be estimated. The parameter estimation method may be interpreted as an optimization method where the minimization of an objective function is sought. Depending on how this function is constructed, the estimation method may be classified as least squares (LS), maximum likelihood (ML), or maximum a posteriori (MAP),[8] which can be either linear or nonlinear and recursive or not. In finite element model updating problems, it is frequent that for the most part the structure be well modeled, whereas localized regions are very poorly modeled, e.g., mechanical joints.[4,5] In such cases, there is generally very little confidence in the initial guess parameter values of the theoretical model and LS or ML estimation methods are suitable. In previous publications, the author has proposed the nonlinear curve fit of dynamic response functions, e.g., frequency response functions (FRF) and unbalanced responses, for such applications.[6,7] It was observed in the various examples treated in those papers that taking the response curves in logarithmic scale improved the convergence of the search procedures involved in the minimization of the objective function. In this paper, it is shown that the minimization of the correlation coefficient between two curves may be obtained in an iterative process where each step consists of a least squares nonlinear fit with a scaled curve. It is also shown that, when the curve is in logarithmic scale, the least squares objective function has the same shape as the maximum correlation objective function, and they are related by a scalar that is the square of the norm of the fitted data vector.

Objective Functions

If F_x is a vector that contains the experimental values of a response function, e.g., a FRF, and F_t is the corresponding vector of theoretically predicted values for the same function, the ordinary least squares objective function J_{LS} is given by,

$$J_{LS}(p) = (F_x - F_t)^T(F_x - F_t) \tag{1}$$

where p is a vector of which the elements are the parameters used in the theoretical model to calculate F_t, and superscript T denotes the transpose of a vector. The objective function for maximizing the correlation coefficient between the two curves may be written as,

$$J_{MC}(p) = 1 - \frac{(F_x^T F_t)^2}{(F_x^T F_x)(F_t^T F_t)} \tag{2}$$

Figures 1 show both objective functions in the case of a one degree-of-freedom (DOF) structure where the absolute value of the FRF was taken as the response function to be fitted.

$$|H(k,m,c,\omega)| = \left|\frac{1}{k - m\omega^2 + i\omega c}\right| \tag{3}$$

In Eq. (3), k is the stiffness coefficient, m the mass, c the damping coefficient of the one-DOF structure, and ω the frequency in radians per second. The curves shown in Figs. 1 are cross sections of the objective functions $J_{LS}(p)$ and $J_{MC}(p)$ in the neighborhood of the solution $p_0 = \{k_0, m_0, c_0\}^T$. The LIN and LOG superscripts are used when the absolute value of the FRF or its logarithm are used, respectively. The curves were scaled for plotting so that the amplitudes would be smaller than 1. It can be seen in Figs. 1 that when the logarithm of the FRF is used the objective functions have the same shape for LS and MC and that this shape is much smoother than for LS with linear FRF amplitudes. The sawtooth profile of the cross sections of $J_{LS}^{LIN}(p)$ explains the poor convergence of the nonlinear search process when the initial guess values of p are not in a close neighborhood of the solution p_0.

Nonlinear Search Algorithm

The Gauss-Newton solution for the minimization of $J_{LS}(p)$ gives the well-known nonlinear LS iterative search algorithm:

$$p^{k+1} = p^k + (S^TS)^{-1}S^T(F_x - F_t) \tag{4}$$

where k denotes the iterative step and S is the sensitivity matrix given by the Jacobian of function F_t with respect to the

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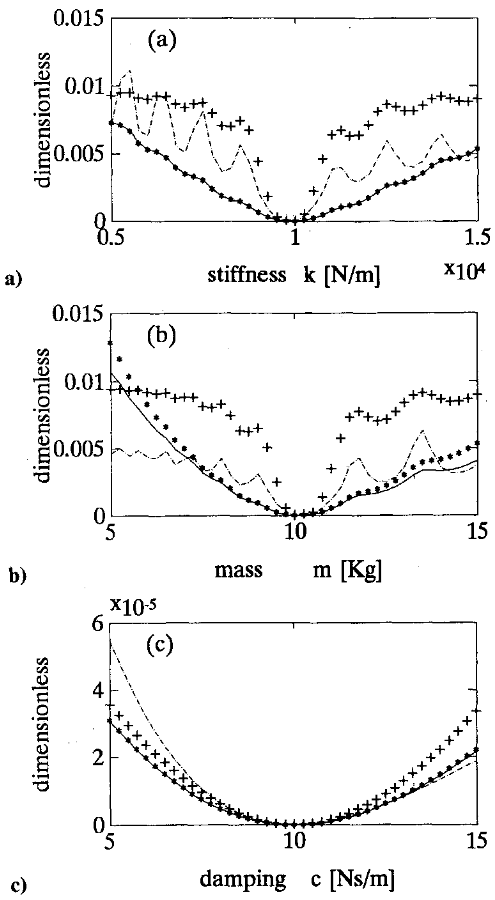

Fig. 1 Objective function cross sections: a) $m = m_0$, $c = c_0$; b) $k = k_0$, $c = c_0$; c) $m = m_0$, $k = k_0$ (-·-·- J_{LS}^{LIN}; —— J_{LS}^{LOG}; + + + J_{MC}^{LIN}; * J_{MC}^{LOG}).**

parameters p, $S = \partial F_t / \partial p$. Linearizing $F_t(p)$ in the neighborhood of p^k gives,

$$F_t(p^{k+1}) \cong F_t(p^k) + S(p^{k+1} - p^k) \tag{5}$$

Replacing Eq. (5) in Eq. (2) and making

$$\left(\frac{\partial J_{MC}(p)}{\partial p}\right)_{p = p^{k+1}} = 0 \tag{6}$$

it is possible to obtain the expression:

$$[F_t^T F_t + 2(p^{k+1} - p^k)^T S^T F_t + (p^{k+1} - p^k)^T S^T S (p^{k+1} - p^k)] S^T F_x - [F_x^T F_t + (p^{k+1} - p^k)^T S^T F_x][S^T F_t + S^T S(p^{k+1} - p^k)] = 0 \tag{7}$$

From Eq. (7), it is possible to write,

$$p^{k+1} = p^k + (S^T S)^{-1} S^T (\bar{F}_x - F_t) \tag{8}$$

where $\bar{F}R_x = \sigma F_x$ is a scaled experimental FRF with

$$\sigma = \frac{F_t^T F_t + 2(p^{k+1} - p^k)^T S^T F_t + (p^{k+1} - p^k)^T S^T S (p^{k+1} - p^k)}{F_x^T F_t + (p^{k+1} - p^k)^T S^T F_x} \tag{9}$$

Therefore, in order to minimize $J_{MC}(p)$, it is necessary to introduce another iterative scheme encompassing the nonlinear least squares iteration loop. Initially, $\sigma = 1$ and the least squares search direction is calculated from Eq. (4). The solution $p^{k+1} - p^k$ is then put in Eq. (9) to calculate σ, which is used to scale F_x. With the scaled curve $\bar{F}_x$, the search direction is recalculated from Eq. (4). This procedure is repeated until convergence of the values of the search direction $(p^{k+1} - p^k)$. When using modified nonlinear LS algorithms,[8] the line search performed in the calculated search direction also uses the objective function given by Eq. (2).

Equivalence of Objective Functions

In this section, the similarity of $J_{LS}^{LOG}(p)$ and $J_{MC}^{LOG}(p)$ in the neighborhood of the solution p_0 will be proved. The LS objective function for logarithmic curves is

$$J_{LS}^{LOG}(p) = \sum_i [\log(F_{t_i}) - \log(F_{x_i})]^2 = \sum_i \left[\log\left(\frac{F_{t_i}}{F_{x_i}}\right)\right]^2 \tag{10}$$

With F_{t_i}/F_{x_i} in the neighborhood of 1, making a Taylor expansion of the logarithm function and neglecting the higher order terms gives,

$$J_{LS}^{LOG}(p) \cong \sum_i \left(\frac{F_{t_i}}{F_{x_i}} - 1\right)^2 \tag{11}$$

The maximum correlation objective function may be written as,

$$J_{MC}^{LOG}(p) = 1 - \frac{[\Sigma_i \log(F_{t_i}) \log(F_{x_i})]^2}{\Sigma_i [\log(F_{t_i})]^2 \Sigma_i [\log(F_{x_i})]^2} \tag{12}$$

Using the Taylor expansion again, it is possible to write,

$$\Sigma_i [\log(F_{t_i})]^2 \Sigma_i \log(F_{x_i})]^2 - [\Sigma_i \log(F_{t_i}) \log(F_{x_i})]^2 \cong \Sigma_i [\log(F_{t_i})]^2 \Sigma_i \left(\frac{F_{t_i}}{F_{x_i}} - 1\right)^2 \tag{13}$$

Substituting Eq. (13) into Eq. (12),

$$J_{MC}^{LOG}(p) \cong \frac{\Sigma_i [(F_{t_i}/F_{x_i}) - 1]^2}{\Sigma_i [\log(F_{t_i})]^2} \tag{14}$$

Comparing Eqs. (14) and (11), the approximate relation between the two objective functions is established:

$$J_{MC}^{LOG}(p) \cong \frac{J_{LS}^{LOG}(p)}{\Sigma_i [\log(F_{t_i})]^2} \tag{15}$$

Equation (15) shows why the objective functions have the same shape near the solution p_0, as can be observed in Figs. 1.

Conclusions

The formulation for a nonlinear curve fit by maximizing the correlation coefficient between the vectors containing the data to be fitted and the corresponding predicted values was derived. The maximization of the correlation coefficient is achieved by iteratively making nonlinear least squares curve fits of scaled curves. It was shown that, in the neighborhood of the solution, the objective functions corresponding to least squares and maximum correlation criteria have the same shape when the curve is fitted in logarithmic scale. This shape is generally much smoother than the linear scale least squares objective function shape. Based on those findings, the proposed technique to improve convergence in nonlinear least squares parameter estimation methods for structural model updating applications consists of curve fitting FRFs in logarithmic scale to obtain a more stable convergence and shifting to linear scale in the last iteration steps to refine the solution.

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